SUPERSONIC NONUNIFORM GAS FLOW AROUND ELONGATED AXISYMMETRIC BODIES

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The equations for a thin viscous shock layer near elongated hyperboloids of revolution oriented at zero angle of attack in a nonuniform gas flow are solved numerically in a wide range of values of the determining parameters of the problem. Two important particular cases of such flows are studied: flow of the far-wake type and flow from a supersonic spherical source. The effect of the type and degree of nonuniformity, the Reynold's number, the shape of the body and the temperature of its surface on the structure of the flow in the viscous layer, the coefficient of friction, and heat transfer factor at the surface are analyzed. It is shown that the effect of the nonuniformity on the parameters of the flow is determined primarily by the shape of the body and the magnitude and direction of the gradient of the total pressure in the incident gas flow, and this effect is of a qualitatively different character near the blunt end of the body and on the side surface for higher values of the longitudinal coordinate. Busemann's formula for the pressure distribution over the body is generalized based on the asymptotic solution of the equations of a thin nonviscous shock layer and a criterion for nondetached nonuniform gas flow around bodies at high Reynolds numbers Re is proposed, it agrees well with the results of numerical calculations.

Flow from a source past a sphere was previously modeled in [1-4], and supersonic flow of the far-wake type around a body was studied in [1, 5-7]. The effect of nonuniformity on the basic characteristics of the flow in the vicinity of the critical point of double curvature was studied in [8]. In [2, 6] the separation of the flow into a nonviscous shock layer and a boundary layer is taken into account, in [1, 3-5] the complete Navier–Stokes equations are solved, and [7, 8] are based on the model of a thin viscous shock layer.

1. Formulation of the Problem. We shall study supersonic nonuniform gas flow around axisymmetric blunt bodies on the basis of the model of a thin viscous shock layer (TVSL). We shall choose a curvilinear coordinate system as follows: the coordinate x is measured along the surface of the body and the coordinate z is measured along the normal to the body. In this coordinate system the TVSL equations have a dimensionless form [8]:

$$
\begin{gathered}
\frac{\partial}{\partial x}(\rho u r)+\frac{\partial}{\partial z}(\rho v r)=0, \quad \frac{\partial p}{\partial z}=0.5(1+\varepsilon)\rho\varkappa u^2,\\
\rho D u=-\frac{2\varepsilon}{1+\varepsilon}\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left(\frac{\mu}{K}\frac{\partial u}{\partial z}\right), \quad p=\rho T, \quad \mu=T^{\omega},\\
\rho D T=\frac{2\varepsilon}{1+\varepsilon}u\frac{\partial p}{\partial x}+\frac{\mu}{K}\left(\frac{\partial u}{\partial z}\right)^2+\frac{\partial}{\partial z}\left(\frac{\mu}{\sigma K}\frac{\partial T}{\partial z}\right), \quad \varepsilon=\frac{\gamma-1}{\gamma+1},\\
D\equiv u\frac{\partial}{\partial x}+v\frac{\partial}{\partial z}, \quad \sigma=\frac{\mu c_p}{\lambda}, \quad \gamma=\frac{c_p}{c_V}, \quad \mathrm{Re}=\frac{\rho_* V_* R}{\mu_0},\\
K=\varepsilon\,\mathrm{Re}, \quad T_0=(\gamma-1)T_*\mathrm{M}_*^2, \quad \mu_0=\mu(T_0),
\end{gathered}
\tag{1.1}
$$

where V_*u and $\varepsilon V_* v$ are the physical components of the velocity vector in the directions x and z, respectively; $\varepsilon^{-1}\rho_*\rho$, $\varepsilon^{-1}p_*T_0p/T_*$, T_0T, and $\mu_0\mu$ are the density, pressure, temperature, and coefficient of viscosity; rR is the distance along the axis of symmetry up to the surface of the body; and, $\varkappa R^{-1}$ is the longitudinal curvature of the contour of the body. All linear dimensions are scaled to R — the radius of curvature of the body at the critical point, and the normal coordinate is scaled to εR.

The system of equations (1.1) is solved with the following boundary conditions:

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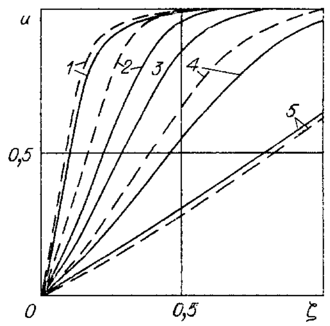

Fig. 1

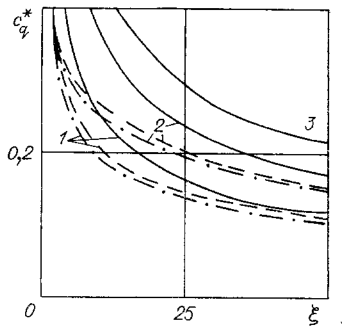

Fig. 2

$$z = 0:\ u = v = 0,\ T = T_w(x); \tag{1.2}$$

$$z = z_s:\ \rho\left(v - u\frac{\partial z_s}{\partial x}\right) = \rho_\infty v_\infty,\quad p = 0.5\,(1+\varepsilon)\,\rho_\infty v_\infty^2,$$
$$\rho_\infty v_\infty (u - u_\infty) = \frac{\mu}{K}\frac{\partial u}{\partial z},\quad \frac{\mu}{\sigma K}\frac{\partial T}{\partial z} = \rho_\infty v_\infty \left\{T - 0.5\left[v_\infty^2 + (u - u_\infty)^2\right]\right\}. \tag{1.3}$$

Here and below the indices ∞, w, and s refer to the values of the variables in the incident flow, on the surface of the body, and on the shock wave; and quantities marked with an asterisk are dimensional values of the parameter along the axis of symmetry in front of the shock wave. Analysis of the Navier–Stokes equations [9] shows that the system (1.1)-(1.3) describes asymptotically correctly the flow in a viscous shock layer with $\varepsilon \ll 1$, $\mathrm{Re} \gg 1$, $K = \varepsilon \mathrm{Re} \geqslant O(1)$.

We shall study below two cases of flow.

1) Flow of the far-wake type around a body. Assuming that the axis of the body is also the axis of symmetry of the wake, the expressions for the components of the velocity vector of the gas and the density of the gas in the incident flow have the following form [1, 6]:

$$u_\infty = V_1 \cos\alpha,\quad v_\infty = -V_1 \sin\alpha,\quad \rho_\infty = [1 + c\,(1 - V_1^2)]^{-1},$$
$$V_1 = (1 - a)^{-1}(1 - a\exp(-br^2)),\ a \ll 1, \tag{1.4}$$

where α is the angle of inclination of the contour of the body with respect to the symmetry axis; the parameters a, b, and c determine the intensity of the wake and depend, according to [6], on the dimensions of the body forming the wake and the distance to it.

2) Flow from a supersonic spherical source past a body. We shall assume that the center of the source lies on the symmetry axis at a distance Rd from the critical point. In a spherical coordinate system we obtain implicit relations between the quantities $u_\infty, v_\infty, p_\infty, \rho_\infty$ [10].

$$u_\infty = V_1\cos(\alpha - \varphi),\ v_\infty = -V_1 \sin(\alpha - \varphi),$$
$$V_1 = \left(\frac{d}{\eta}\right)^2 A^{-\frac{1}{\gamma-1}},\quad \rho_\infty = A^{1/(\gamma-1)},\quad \sin\varphi = \frac{r}{\eta}, \tag{1.5}$$
$$A = 1 + 0.5\,(\gamma - 1)\,M_*^2\,(1 - V_1^2),\quad M_*^2 = \frac{V_*^2\rho_*}{\gamma p_*}$$

(ηR is the distance from the center of the source). The nonuniformity of the flow is determined by the parameters d and M_*, one of which can be replaced by giving the radius of the source.

2. Numerical Solution of the Problem. For convenience in solving the problem numerically we shall transform the starting system of equations and the boundary conditions of A. A. Dorodnitsyn's type: These variables permit resolving the singularities at the critical point:

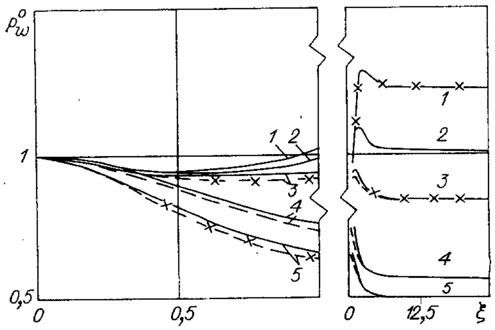

Fig. 3

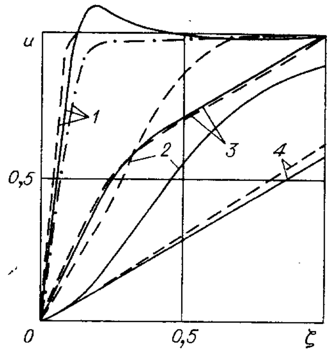

Fig. 4

$$\xi = x,\quad \theta = \frac{2T}{v_\infty^2},\quad \zeta = \frac{1}{\Delta}\int\limits_0^z \frac{\rho r}{\xi}\,dz,\quad \Delta = \int\limits_0^{z_s} \frac{\rho r}{\xi}\,dz,$$
$$u_0 = \frac{u}{u_\infty} = \frac{\partial f}{\partial \zeta},\quad P_1 = \frac{1}{\xi}\frac{\partial p}{\partial \xi},\quad l = \frac{\mu\rho r^2}{\Delta^2 K^2 \xi^2}. \tag{2.1}$$

In the variables (2.1) the initial and boundary value problem (1.1)-(1.3) assumes the following form (we omit the index 0):

$$(lu_\zeta')_\zeta' = Du + \alpha_1 u^2 + \alpha_2 \rho^{-1} P_1,\quad p_\zeta' = \beta_1 u^2,$$
$$(l\sigma^{-1}\theta_\zeta')_\zeta' = D\theta + \alpha_3 u\theta - \alpha_4 l\,(u_\zeta')^2 - \alpha_5 u\rho^{-1} P_1, \tag{2.2}$$
$$(P_1)_\zeta' = \beta_2 u u_\xi' + \beta_3 u^2,\quad D \equiv u_\infty \frac{\partial}{\partial \xi} - (u_\infty u f_\xi' + \alpha_0 f)\frac{\partial}{\partial \zeta},\quad p = 0.5 v_\infty^2 \rho\theta;$$
$$\zeta = 0:\ u = f = 0,\ \theta = \theta_w; \tag{2.3}$$
$$\zeta = 1:\ l\Phi u_\zeta' + u = 1,\quad l\sigma^{-1}\Phi\theta_\zeta' + \theta = 1 + 0.5\alpha_4 (u-1)^2,$$
$$\alpha_0 f + u_\infty f_\xi' = \Phi^{-1},\quad p = \frac{1+\varepsilon}{2}\rho_\infty v_\infty^2,\quad \Phi = -\frac{\Delta\xi}{r\rho_\infty v_\infty}, \tag{2.4}$$
$$P_1 = P_{1s} = (1+\varepsilon)\left[\frac{v_\infty^2}{2\xi}\frac{\partial\rho_\infty}{\partial\xi} + \frac{\rho_\infty v_\infty}{\xi}\frac{\partial v_\infty}{\partial \xi}\right].$$

The coefficients α_0-α_5 amd β_1-β_3 of the system (2.2)-(2.4) are functions of the longitudinal coordinate ξ and depend as follows on the geometry of the body and the parameters of the incident flow:

$$\alpha_1 = \frac{\partial V_1}{\partial \xi}\cos(\alpha - \varphi) + (\varkappa + \varkappa_1) V_1 \sin(\alpha - \varphi),\quad \varkappa_1 = \frac{\partial\varphi}{\partial\xi},$$
$$\alpha_0 = \alpha_1 + u_\infty \frac{\partial \ln \Delta}{\partial \xi} + a_2 V_1,\quad \alpha_2 = \frac{2\varepsilon}{(1+\varepsilon)\,a_2 V_1},\quad a_1 = \frac{r}{\xi},$$
$$\alpha_3 = 2\frac{\partial v_\infty}{\partial \xi}\operatorname{ctg}(\alpha - \varphi),\quad \alpha_4 = 2\operatorname{ctg}^2(\alpha - \varphi),\quad \alpha_5 = \alpha_2\alpha_4, \tag{2.5}$$
$$\beta_1 = 0.5(1+\varepsilon)\varkappa\Delta a_1^{-1} V_1^2 \cos^2(\alpha - \varphi),\quad \beta_2 = 2\beta_1\xi^{-1},\quad \beta_3 = \frac{1}{\xi}\frac{\partial\beta_1}{\partial\xi},$$
$$\frac{\partial v_\infty}{\partial \xi} = V_1(\varkappa + \varkappa_1)\cos(\alpha - \varphi) - \frac{\partial V_1}{\partial \xi}\sin(\alpha - \varphi),\quad a_2 = \xi^{-1}\cos(\alpha - \varphi).$$

For the case 1 the substitution $\varphi = \varkappa_1 = 0$ must be made in the formulas (2.5), and in the case 2 the value of φ is determined from (1.5).

Analysis of (2.2)-(2.5) shows that for the chosen scaling the effect of the nonuniformity of the flow on the characteristics of the flow near the critical point in the case 1 is determined solely by one parameter $\Lambda = 2ab(1 + c)/(1 - a)$, which appears only in the boundary condition for the gradient of the pressure on the shock wave. For the case 2 this effect, generally speaking, is determined by the two parameters d and M_*, but for sufficiently high supersonic velocities of the flow around the body the effect of M_* can be neglected, so that the only remaining parameter characterizing the degree of nonuniformity of the flow is d. The parameter d appears both in the boundary conditions on the shock wave and in the coefficients of the equations themselves.

The system (2.2)-(2.4) was solved numerically based on an implicit (in the coordinate ζ) finite-difference scheme with accuracy $O(\Delta\zeta)^4 + O(\Delta\xi)^2$; this scheme consists of the scheme employed in [11], where the method of [12], which combines second-order accuracy with good stabilizing properties, was employed to approximate the derivatives with respect to ξ. Flow around hyperboloids of revolution with the following parameters of the problem were studied: $\varepsilon = 0.1$, $\sigma = 0.71$, $\omega = 0.5$, $c = 3$, $M_* = 10$, $d \geqslant 1$, $10 \leqslant \mathrm{Re} \leqslant 5 \cdot 10^5$, $0.03 \leqslant \theta_w = \mathrm{const} \leqslant 0.5$, $10° \leqslant \beta \leqslant 45°$, $0.01 \leqslant b \leqslant 20$, $0 \leqslant \Lambda \leqslant \Lambda_* = 4/3$.

3. Discussion of Computational Results. Case 1. Analysis shows that for the case of flow on the side surface the effect of nonuniformity on the flow depends on, in addition to the parameter Λ, the characteristic radius of the wake $r_* \sim b^{-1/2}$. The form of this effect is largely determined by the relative values of Λ, r_*, and Re.

In particular, for fixed Λ, large b (small r_*), and any values of Re the effect of the degree of nonuniformity is localized primarily in a neighborhood of the blunt end, so that for $\xi \gg 1$ the velocity and temperature profiles are virtually identical to those for uniform flow around the body. At the same time for sufficiently large r_* this effect is largely determined by Re. This result is illustrated well in Fig. 1, which shows the profiles of u across the shock layer with $\theta_w = 0.25$, $\beta = 45°$, $\mathrm{Re} = 10$, $\xi = 50; 25; 5; 0$ (lines 2-5) and $\mathrm{Re} = 10^5$, $\xi = 5$ (line 1) for a uniform flow ($\Lambda = 0$) (solid lines) and $\Lambda = 2/3$, $b = 1$ (dashed lines). One can see that for sufficiently large values of Re the difference in the profiles can already be neglected for $\xi \gtrsim 5$, but for $\mathrm{Re} \approx 10$ the effect of nonuniformity remains significant also for $\xi \gtrsim 50$.

The dependence of the distribution of the heat flux $c_q = (\mu\partial T/\partial z)/(\varepsilon\sigma\sqrt{\mathrm{Re}})$ along the surface on the parameters Λ and b is shown in Fig. 2, where the values of $c_q^* = c_q(\xi, \Lambda)/c_q(0, 0)$ are given for $\mathrm{Re} = 10$, $\theta_w = 0.25$ (solid lines), $\mathrm{Re} = 10^5$, $\theta_w = 0.25$ (dashed lines), and $\mathrm{Re} = 10^5$, $\theta_w = 0.03$ (dot-dashed lines) with $\Lambda = 0$; $\Lambda = 2/3$, $b = 1.5$; $\Lambda = 2/3$, $b = 1.2$ – the lines 1-3. On the whole the calculations have shown that the behavior of c_q^* depends strongly on the intensity of the wake, the radius, and the longitudinal coordinate ξ. Near the bluntend at $\xi = O(1)$ an increase in Λ results in a decrease in the absolute values of the heat flux. For small radii of the wake the maximum of c_q can be shifted from the critical point, and its location correlates quite well with the boundary of the wake. On the side surface for sufficienty large ξ an increase in the degree of nonuniformity, conversely, results in an increase in the absolute values of c_q as compared with the case of uniform flow around the body, and in addition for large b this excess is in practice insignificant, whereas for $b \approx 1$ it is quite significant. It should also be noted that as the degree of nonuniformity increases the effect of the temperature of the surface and Re on the distribution of c_q^* increases.

It is shown in [1, 5, 6] that because of the existence of a positive gradient of the total pressure in the incident flow a zone of return-circulation flow can appear in the shock layer near the critical point. In [6, 8] the condition is proposed as a criterion for the appearance of such a zone at $\xi = 0$ for $\mathrm{Re} \to \infty$.

$$\partial^2 p_w/\partial\xi^2 = 0. \tag{3.1}$$

Using the asymptotic approach of Hayes–Busemann [13] to determine p_w we find that there is no flow detachment at the critical point if

$$\Lambda \leqslant \Lambda_* = 4/3. \tag{3.2}$$

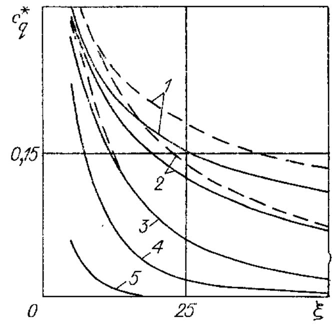

Fig. 5

For a number of bodies, for example, spheres, the condition (3.2) remains valid also for flow outside the neighborhood of the critical point. At the same time there exist entire classes of bodies for which the condition $\Lambda < \Lambda_*$ is not a sufficient condition of undetached flow around the body, since for such bodies the pressure gradient at the surface can become positive not at the critical point but rather for $\xi > 0$. This is connected with the fact that $\partial p_w/\partial\xi$ depends both on the gradient of the total pressure in the incident flow and on $\varkappa$ – the curvature of the shock wave. In a neighborhood of the critical point for blunt bodies $\varkappa = O(1)$, as a result of which the effect of the second factor on the distribution of $\partial p_w/\partial\xi$ is significant and detachment of the flow does not occur. For $\xi \gg 1$, however, for some bodies, for example, for hyperboloids, $\varkappa \to 0$, so that the effect of the gradient of the total pressure, which, as already noted above, is positive for the wake, can be determining. Thus it may be concluded that the appearance of detachment of flow on the side surface of the body is determined by the ratio of the orders with which the longitudinal curvature of the surface of the body and the positive gradient of the total pressure in the incident flow approach zero. In particular, for fixed Λ and small radii of the wake (large b) the degree of nonuniformity of the flow approaches zero more rapidly and $p_w(\xi)$ is a monotonically decreasing function under these conditions. As r_* increases, however, the situation changes – the distribution $p_w(\xi)$ ceases to be monotonic and a characteristic "spoon" of pressure arises on the side surface. This can be seen clearly in Fig. 3, which shows the dependences $p_w^0 = p_w(\xi)/p_w(0)$ for $\beta = 45°$, Re = 10 (solid lines), Re = 10^5 (dashed lines) with $\Lambda = 0$ (line 5), $\Lambda = 2/3$, b = 1; 1.2; 1.5; 5 (lines 1-4). On the whole the pressure along the surface is virtually independent of Re, but for regimes in which a region with a positive pressure gradient arises on the surface of the body an increase in Re can cause detachment of the flow. Based on this we conclude that as Re $\to \infty$ the equation

$$\partial p_w/\partial\xi < 0 \tag{3.3}$$

for all points on the side surface is a sufficient condition for undetached flow around the body. As comparisons with numerical calculation show (crosses in Fig. 3) the quantity p_w can be calculated with good accuracy from the formula obtained by analogy to Busemann's formula:

$$p_w = \frac{1+\varepsilon}{2}\left[\rho_\infty V_1^2 \sin^2\alpha - \frac{\varkappa}{r_w}\int_0^{r_w} r\rho_\infty V_1^2\cos^2\alpha\, dr\right]. \tag{3.4}$$

Case 2. Figure 4 gives an idea of the character of the effect of the distance d from the source on the structure of the flow in the shock layer. The figure shows the profiles of u across the layer with $\beta = 45°$, $\theta_w = 0.25$ for a uniform flow (dashed lines), d = 10 (dot-dashed lines), and d = 3 (solid lines) with Re = 10 (lines 2 and 4), Re = 10^3 (lines 1 and 3), and ξ = 0 and 25 (lines 3, 4, and 1, 2). One can see that as for the case 1 the effect of nonuniformity is a function not only on d but also Re and the longitudinal coordinate. For low values of Re the corresponding profiles of u in a neighborhood of the critical point are close to one another, but on the side surface the difference can reach 40-50%. At the same time for Re ≫ 1 the difference remains comparatively small for any value of ξ, though unlike the case of uniform flow around the body with sufficiently small values of d and $\xi \gtrsim 10$ characteristic local maxima near the surface of the body appear in the profiles of u and θ.

On the whole the effect of nonuniformity on the flow in the case of a body in a flow from a supersonic source is of a qualitatively different character than in the case of a body in a flow of the waste type. In particular, in this case the "spoon" of pressure does not appear in the distribution p_w, and $p_w(\xi)$ is always a monotonically decreasing function, while detachment of the flow can occur only for small values of d, owing to the appearance of a point with zero pressure on the surface of the body.

In conclusion we shall discuss the question of the dependence of the parameter d of the distributions of the heat flow c_q along the surface. Calculations showed that in a neighborhood of the critical lines, decreasing d decreases the thickness of the shock layer and increases c_q. For not very small values of d it is possible to find an approximation formula that enables calculation of the value of c_q based on the corresponding value of $c_q{}^0$ for a body in a uniform flow:

$$c_q(d, \mathrm{Re}) = \sqrt{1 + d^{-1}}\, c_q^0(\infty, \mathrm{Re}^0). \tag{3.5}$$

Here $c_q{}^0(\infty, \mathrm{Re}^0)$ is the heat flux into the critical point of this body placed in a uniform flow of gas with $\mathrm{Re}^0 = d\,\mathrm{Re}/(1 + d)$.

Comparison with the results of numerical calculations for a cooled surface ($\theta_w < 0.2$) shows that the accuracy of the formula (3.5) with $d \gtrsim 1$ is of the order of 5% in the entire range of Reynold's numbers. As the distance from the critical point increases the character of the dependence of c_q on d changes. Figure 5, where the distributions c_q refer to the corresponding values for a body in a uniform flow at the critical point with $\mathrm{Re} = 10^5$, $\beta = 45°$, $d = 10^5$; 10^2; 25; 10; 3 (lines 1-5) for $\theta_w = 0.25$ (solid lines) and $\theta_w = 0.03$ (dashed lines), are presented, shows that unlike near the critical point a decrease in d for sufficiently large values of ξ results in a decrease in the absolute values of the heat flux. It is important to note that this must be taken into account in the calculation of flow around elongated bodies, even if the small degree of nonuniformity is quite small (d is large). For example, for d = 100 for $\xi \leq 5$ the value of c_q differs from the value for a uniform flow by not more than 0.5%, whereas for $\xi = 50$ this difference already equals 35%.

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FLOW IN A CHANNEL WITH SUCTION ON ONE SIDE: DETACHMENT FROM AN IMPERMEABLE WALL AND EFFECT OF ROTATION AROUND THE TRANSVERSE AXIS

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1. Plane Flow: Review of Formulations and Results. We shall study the plane flow of a viscous incompressible liquid along a channel formed by two parallel walls, under conditions such that one is impermeable and liquid is suctioned uniformly through the other wall. We denote the distance between the walls by H. We shall assume that the flow occurs in the y0z plane; we place the origin of the Cartesian coordinate system in the inlet section on the impermeable wall and we orient the z-axis parallel to the walls in the direction of the flow. The equations of motion and continuity have the form

$$w\frac{\partial w}{\partial z}+v\frac{\partial w}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial y^2}\right),$$
$$w\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial y^2}\right),\quad \frac{\partial w}{\partial z}+\frac{\partial v}{\partial y}=0. \tag{1.1}$$

The solution of the equations must satisfy the boundary conditions

$$y=0:\quad w=v=0;\qquad y=H:\quad w=0,\ v=v_s \tag{1.2}$$

($v_s > 0$ is the suction velocity).

In [1, 2] it is shown that the system (1.1) can have a self-similar solution of the form

$$w=(W_m-v_s z/H)f'(\eta),\ v=v_s f(\eta). \tag{1.3}$$

Here W_m is the mean flow at the inlet; $\eta = y/H$. For the problem at hand the function $f(\eta)$ satisfies the equation

$$f'''+\mathrm{R}_s\left(f'^2-ff''\right)=k \tag{1.4}$$

and the boundary conditions

$$f(0)=f'(0)=0,\ f(1)=1,\ f'(1)=0. \tag{1.5}$$

One of the boundary conditions is used to find the constant k which determines the longitudinal pressure gradient. The parameter of the self-similar solution is Reynolds number $\mathrm{R}_s=v_sH/\nu$, contructed based on the suction velocity. In [3] the problem (1.4) and (1.5) is solved analytically by the method of expansion in a series in powers of R_s and the solution is valid for small values of this parameter. We do not know of any other solutions of the self-similar problem.

The solutions of the two-dimensional problem (1.1) and (1.2) were found by the finite-difference method in [4]. The purpose of the calculations was to study the flow field in a flat heat pipe. The downstream end of the pipe was assumed to be closed. The conditions at the inlet into the suctioned section (condenser section) were not fixed; they were determined

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